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Acoustic Streaming in the Cochlea

Simulation of a Fluid-Structure Coupled System

Introduction

The existence and relevance of Acoustic Streaming for the mechanical processes in hearing has remained controversal. Main contributions to this topic came from the Bell Telephone Laboratories in 1972, the work of Tonndorf and later from Lighthill. Today a general assumption is the relevance of Acoustic Streaming to be limited to very high sound pressure levels larger than 120dB (SPL). Because of the unaccessability of the organ of Corti in the inner ear an experimental proof was not possible up to now, except the pioneering work of G. v. Békésy who saw eddies in the lymph of the cochlea with indeed high stimulating pressure levels (140dB (SPL)). Therefore the actual opinion of Acoustic Streaming in the ear ranges from it's irrelevance to it's permanent presence.
Acoustic streaming is a physical phenomenon which was first proposed by Lord Rayleigh. It denotes sound induced flow (streaming) in fluids, e.g. air or water. Later it was extended by a "boundary drive" mechanism [1]. Köster [2] applies these theories to micro-fluidic mixing devices and provides a software for numerical evaluations, but the specific anatomical, physical and physiological properties of the cochlea with simultaneous consideration of the fluid-structure interaction was not yet implied. Rayleigh's law of streaming is applicable to two kinds of streaming motions

1. Acoustic Streaming, resulting when an acoustic standing wave in a fluid adjacent to a solid wall suffers dissipation within the resulting boundary layer and
2. a related kind of streaming which results from the vibrations of a solid body adjacent to fluid at rest.

Because the sound dissipation, the sound generation and the wave propagation in the cochlea of the inner ear of humans and animals are unsolved problems up to now we develope a mathematical theory which implements the fluid-structure interaction of the complex biomechanical system for a numerical evaluation.

Acoustic Streaming

The system of equations that describes the fluid flow consists of the conversation relations of mass and momentum with the velocity $\boldsymbol{v}(\boldsymbol{x}, t)$, pressure $p(\boldsymbol{x}, t)$, density of the fluid $\rho(\boldsymbol{x}, t)$, shear viscosity μ and bulk viscosity μ_B

$$\nabla p + \mu\nabla^2\boldsymbol{v} + (\mu_B + \frac{\mu}{3})\nabla\nabla\cdot\boldsymbol{v} = \rho\ \frac{\partial\boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v}) \tag{1}$$

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\boldsymbol{v}) = 0 \tag{2}$$

and the constitutive relation $\rho = \rho(p)$. At the boundaries the no-slip condition is assumed. Since the acoustic streaming problem is a nonlinear effect the usual perturbation expansion of the unknowns $\boldsymbol{v}$, p and ρ is performed:

$$\begin{aligned} \boldsymbol{v} &= \boldsymbol{0} + \boldsymbol{v}^{(1)} + \boldsymbol{v}^{(2)} + \mathcal{O}(\epsilon^3) \\ p &= p^{(0)} + p^{(1)} + p^{(2)} + \mathcal{O}(\epsilon^3) \\ \rho &= \rho^{(0)} + \rho^{(1)} + \rho^{(2)} + \mathcal{O}(\epsilon^3) \end{aligned} \tag{3}$$

By combining equations (1), (2) and (3) and gathering only the linear terms the equations of mass and momentum become

$$\nabla p^{(1)} + \mu\nabla^2\boldsymbol{v}^{(1)} + (\mu_B + \frac{\mu}{3})\nabla\nabla\cdot\boldsymbol{v}^{(1)} = \rho^{(0)}\frac{\partial\boldsymbol{v}^{(1)}}{\partial t} \tag{4}$$

$$\frac{1}{c_0^2\rho^{(0)}}\frac{\partial p^{(1)}}{\partial t} + \nabla\cdot\boldsymbol{v}^{(1)} = \boldsymbol{0} \tag{5}$$

where $p^{(1)} = c_0^2\rho^{(1)}$ is assumed. The first order system is the standard linear system which describes the damped propagation of sound in a viscous fluid.
The solutions of the linear velocity, pressure and displacement field are used for the time-averaged second order system of equations of mass and momentum, given by

$$\begin{aligned} \nabla p^{(2)} \quad \mu\nabla^2\boldsymbol{v}^{(2)} \quad (\mu_B + \frac{\mu}{3})\nabla\nabla\cdot\boldsymbol{v}^{(2)} &= \\ \frac{1}{c_0^2}\langle p^{(1)}\frac{\partial\boldsymbol{v}^{(1)}}{\partial t}\rangle \quad \rho^{(0)}\langle(\boldsymbol{v}^{(1)}\cdot\nabla)\boldsymbol{v}^{(1)}\rangle \end{aligned} \tag{6}$$

$$\rho^{(0)}\nabla\cdot\boldsymbol{v}^{(2)} = \quad \frac{1}{c_0^2}\nabla\cdot\langle p^{(1)}\boldsymbol{v}^{(1)}\rangle \tag{7}$$

where $\langle\cdot\rangle$ denotes the temporal average of a time-dependent function. Note, that the second harmonic terms are also time-averaged, since the time-independent part of the fluid-motion is of interest.

Basilar Membrane

To simulate acoustic streaming within the cochlea the macromechanics of the basilar membrane have to be taken into account. Numerous models - describing these mechanics - were developed by several researchers. Mammano [3] proposes to use the following linear differential equation

$$m(x)\frac{\partial^2 u}{\partial t^2} + h(x)\frac{\partial u}{\partial t} + \quad \frac{\partial}{\partial x}s(x)\frac{\partial}{\partial x})\frac{\partial u}{\partial t} + k(x)u = f(x) \tag{8}$$

where x describes the longitudinal position and $u(x, t)$ the vertical deflection of the basilar membrane. The coefficient-functions $m(x)$, $h(x)$, $s(x)$ and $k(x)$ describe the local mass, the fluid viscosity, the shearing resistance and the stiffness.
Another equation governing the deflection of the basilar membrane - as recommended by Böhnke [4] - is given by

$$\rho_P h\frac{\partial^2 u}{\partial t^2} + \frac{E_x h^3}{12b(x)}\frac{\partial^2}{\partial x^2}\left(b(x)\frac{\partial^2 u}{\partial x^2}\right) + r\frac{\partial u}{\partial t} + \frac{2E_y h^3}{b^4(x)}u \quad p_e = 0 \tag{9}$$

where ρ_P, h, $b(x)$, r and p_e denote the density, the mean thickness, the width, friction and the external pressure. This differential equation is based upon the beam theory which is used to describe the bony fibers lying transverse to the longitudinal direction. The bony fibers described by the Young's modulus E_y are embedded in a rather soft tissue described by the Young's modulus E_x.

Fluid Structure Coupling

The boundary condition that describes the coupling between the fluid and the structure is given by

$$\boldsymbol{v} = \frac{\partial u}{\partial t} \tag{10}$$

Applying a finite element discretisation of the acoustic system (4) and (5) and the equation of motion of the basilar membran (8) or (9) with simultaneous consideration of the coupling condition (10) yields

$$\boldsymbol{V}\ddot{\tilde{\boldsymbol{x}}} + \boldsymbol{D}\dot{\tilde{\boldsymbol{x}}} + \boldsymbol{G}\tilde{\boldsymbol{x}} = \boldsymbol{F} \tag{11}$$

where $\tilde{x}$ is the discretized vector of the unknown functions $\boldsymbol{v}(\boldsymbol{x}, t)$, $p(\boldsymbol{x}, t)$ and $u(x, t)$. The matrices $\boldsymbol{V}$, $\boldsymbol{D}$, $\boldsymbol{G}$ and the load Vector $\boldsymbol{F}$ can be assembled by the discretization process. Finally the acoustic streaming field can be derived by an appropriate finite element discretisation of the second order system (6) and (7).

Goals

This work is motivated by the knowledge that already small deflections of the inner- and outer-haircells - caused by a fluid flow - have a significant effect on the active amplification process and on the neuronal stimulation.
Within the scope of the numerical evaluation we want to discuss the issue of how the acoustic streaming has influence on the micro- and macro-mechanical functionality of the cochlea. The answer to this question is important for future modeling and for a better understanding of how the cochlea works.

References

[1] C.E. Bradley. Acoustic streaming field structure: The influence of the radiator. Journal of the Acoustical Society of America, 100(3), (1996), 1399-1408

[2] D. Köster. Numerical Simulation of Acoustic Streaming on SAW-driven Biochips. PhD thesis, Mathematisch-Naturwissenschaftliche Fakultät der Universität Augsburg (2006), 1-148

[3] F. Mammano and R. Nobili. Biophysics of the cochlea: Linear approximation. Journal of the Acoustical Society of America, **93**(6), (1993), 3320–3332

[4] F. Böhnke. Wellenausbreitung auf der Basilarplatte des menschlichen Ohres. PhD thesis, Falkultät für Technische Wissenschaften der Technischen Universität Ilmenau, (1991)